

Rosen-Zener interferometry with Ultracold Atoms

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We propose a time-domain "interferometer" based on ultracold Bose atoms loaded on a double well potential. By the adiabatic Rosen-Zener process, the barrier between two wells is ramped down slowly, held for a while, then ramped back. Starting with a coherent state of double well system, the final occupations on one well show interesting interference fringes in the time-domain. The fringe pattern is sensitive to the initial state, the interatomic interaction, and the external forces such as gravity which can change the shape of the double well. In this sense, this interferometric scheme has the potentials for precision measurements with ultracold atoms. The underlying mechanism is revealed and possible applications are discussed.

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Quantum interference is one of the most fundamental and challenging principles in quantum mechanics, and has various applications in high-precision measurement and quantum coherent control [1, 2, 3]. Recently, a number of quantum interference experiments have been performed with Bose-Einstein condensates (BECs) [4, 5, 6], where the coherent matter wave serves as phase coherent sources. Using the device with matter waves instead of photons can improve the measurement precision by a factor of 10^4 as shown in [7, 8], and the limit of phase sensitivity of standard interferometer can be surpassed by matter wave interferometers [9]. In particular, well-developed techniques in preparing and manipulating BECs in the double well brought a new research surge [10, 11, 12, 13, 14]. All these experiments demonstrated the macroscopic quantum coherence of double well BECs with a spatial interferometer.

In this letter, we propose an interferometer realized by the adiabatic Rosen-Zener process with double-well BECs. The Rosen-Zener model, proposed by Rosen and Zener to account for the spin-flip of two-level atoms interacting with a rotating magnetic field in Stern-Gerlach experiments [15]. Here, in a double well scheme, the adiabatic Rosen-Zener process is performed by slowly lowering the barrier of the double well to a height and holding at this height, then adiabatically lifting it back to the original height. Due to inter-atom interaction, the system is an intrinsic nonlinear system. Through such a nonlinear Rosen-Zener process, the system shows marvelous interference effects in the time domain.

We consider a Bose atomic condensate trapped in a double well potential with strongly transverse confinement. The dynamics obey a 1D model

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = H_0 \Psi(x, t) + \lambda |\Psi(x, t)|^2 \Psi(x, t), \quad (1)$$

where $H_0 = -(\hbar^2/2m)(\partial^2/\partial x^2) + V(x)$, $\lambda =$

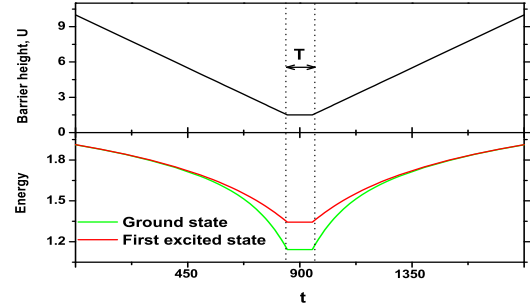


FIG. 1: Schematic diagram for an adiabatic Rosen Zener process. Upper panel: the barrier height between two wells in time sequence. Bottom panel: the energies of the Ground state and the first excited state in time with varying barrier.

$8N\pi\hbar^2\beta_{1d}a_s/m$, m is the single-atom mass and a_s is the s-wave scattering length describing the inter-atom interaction, N is the total particle number, and β_{1d} is the compensating coefficient for reducing transverse freedoms. $V(x)$ is a double well potential realized by superposing a Gaussian barrier (see Fig. 1) on a harmonic trap

$$V(x) = \frac{1}{2}\omega^2 x^2 + U \exp\left(-\frac{x^2}{2d^2}\right), \quad (2)$$

in which ω is the trapping frequency of the harmonic potential, d is the barrier width, and U is the barrier height. In the adiabatic Rosen-Zener process, we ramp the barrier height as shown in Fig 1. The barrier is lowered slowly to a proper height, and holding for time interval T , ramped up again slowly.

For a BEC trapped in a double well, when the barrier is high enough, the ground state (GDS) $|\Psi_g\rangle$ and the

first excited state (FES) $|\Psi_e\rangle$ are degenerate. The system has two local stable modes that are superpositions of the ground state and the first excited state, namely, $|\Psi_L\rangle = \frac{1}{\sqrt{2}}(|\Psi_g\rangle - |\Psi_e\rangle)$ and $|\Psi_R\rangle = \frac{1}{\sqrt{2}}(|\Psi_g\rangle + |\Psi_e\rangle)$. For the two local stable modes, almost all the atoms are localized in a single well, which are well-known as self-trapping states [16, 17, 18].

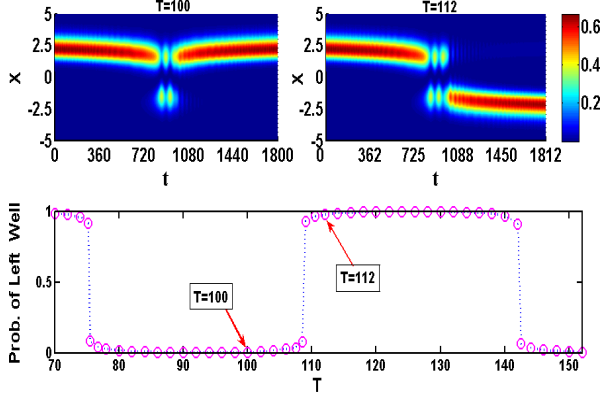


FIG. 2: Coherent transition of atoms by the Rosen Zener process. The upper figures give two example of such a process. For the holding time $T = 100$ (left figure), atoms are still in the initial well, however, for $T = 112$ (right one) almost all atoms are transferred to another well. The lower panel shows the probabilities of occupation on the left well for different holding times T (all atoms in right well initially)

Throughout the entire process, the barrier is changed very slowly so that the excitations to high eigenstates are very small. Hence, we expect the atoms still occupy only on the ground state and the first excited state at final, $|\Psi_f\rangle = c_0|\Psi_g\rangle + c_1|\Psi_e\rangle$. From the definition of the local model, we have $|\Psi_f\rangle = a|\Psi_L\rangle + b|\Psi_R\rangle$ with $a = \frac{1}{\sqrt{2}}(c_0 - c_1)$ and $b = \frac{1}{\sqrt{2}}(c_0 + c_1)$, in which $|c_0|^2 + |c_1|^2 = 1$. Therefore, the final probabilities on the left and right well are

$$|a|^2 = \frac{1}{2} - |c_0||c_1|\cos\theta, \quad |b|^2 = \frac{1}{2} + |c_0||c_1|\cos\theta, \quad (3)$$

where $\theta = \arg c_1 - \arg c_0$ is the relative phase between $|\Psi_g\rangle$ and $|\Psi_e\rangle$. The final occupations on each well are coherent interference of the ground state and the first excited state, and depend on the relative phase and probabilities of the two state. The final occupation of one well serves as the interferometer "output ports".

First we consider when the BECs are initially localized in one well (e.g., the right one) and the barrier between two wells is sufficiently high. The dynamics of BECs in the designed Rosen-Zener process is demonstrated by Fig. 2. The results are obtained by directly solving the GP equation using the operator-splitting approach for the following dimensionless parameters: the trapping frequency $\omega = 0.2\pi$, the barrier width $d = 1/\sqrt{2}$, the

initial and final height $U_0 = U_f = 10$, the lowest height $U_h = 1.5$, and the ramping rate $\alpha = 0.01$. The upper two figures show the density evolution during the Rosen-Zener process with the holding time $T = 100$ and $T = 112$, respectively. During the holding time, the Josephson oscillations are clearly visible. Finally, all atoms completely localize in one well, and which well is occupied depends on the holding time. Moreover, the process is robust, that is, there exist several intermittent windows of complete transfer and blockade in the holding time (see the bottom panel of Fig.2). The period of the rectangle functions is determined by the atomic interaction and tends to infinity at certain interactions as will be shown latter.

We assume the system only occupies the ground and first excited states during the adiabatic process, i.e.,

$$\Psi(x, t) = c_0(t)\Psi_g(x, U) + c_1(t)\Psi_e(x, U) \quad (4)$$

where $\Psi_g(x, U)$ and $\Psi_e(x, U)$ are the ground and first excited states for GP equation (1) with the barrier height U , which obey $E_j\Psi_j = H_0\Psi_j + \lambda\Psi_j^3$, where E_j is the chemical potential for Ψ_j ($j = e, g$). Defining $z = |c_1|^2 - |c_0|^2$ and $\theta = \arg(c_1) - \arg(c_0)$, we obtain the equations for z and θ from GP equation (1): $\dot{z} = -\frac{\partial H}{\partial \theta}$, $\dot{\theta} = \frac{\partial H}{\partial z}$ with the classical Hamiltonian

$$H = \delta z + \frac{\beta}{2}z^2 + \frac{\Lambda}{2}(1 - z^2)\cos 2\theta, \quad (5)$$

in which $\delta = E_e - E_g + \frac{1}{2}(\gamma_{gg} - \gamma_{ee})$, $\beta = \frac{1}{2}(\gamma_{ee} + \gamma_{gg} - 2\gamma_{eg})$, $\Lambda = \gamma_{eg}$, and $\gamma_{ij} = \lambda \int \Psi_i^2 \Psi_j^2 dx$ ($i, j = e, g$). In the above deductions, the integrals with odd powers of Ψ_e and Ψ_g are nearly zero and have been omitted[19]. In the upper panel of Fig. 3, we plot $E_e - E_g$, γ_{ij} as functions of the barrier height U .

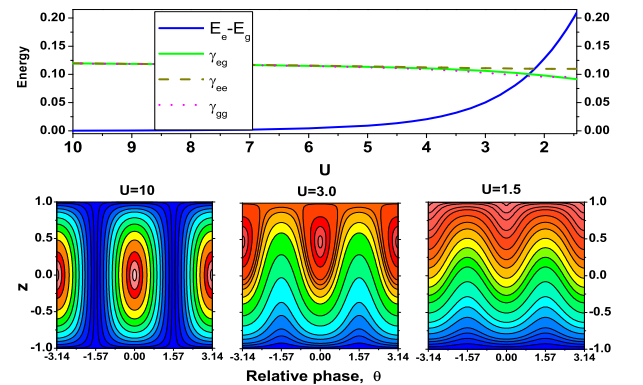


FIG. 3: The upper panel is for γ_{gg} , γ_{eg} , γ_{ee} , and $E_e - E_g$ for different barrier heights with the same parameters for Fig. 2. The bottom row shows three phase spaces of the classical Hamiltonian (5) for $U = 10, 3, 1.5$ respectively.

The above classical Hamiltonian system has two axial fixed points at $z = \pm 1$ independent of the relative phase, in which $z = -1$ corresponds to the ground state of GP equation (1) and $z = +1$ corresponds to the first excited state. The other fixed points of the above classical Hamiltonian can be obtained by solving the equation $\dot{z} = 0$ and $\dot{\theta} = 0$. For $\delta < (\Lambda - \beta)$, we obtain $(z^*, \theta^*) = (\frac{\delta}{\Lambda - \beta}, 0)$ and $(z^*, \theta^*) = (\frac{\delta}{\Lambda - \beta}, \pi)$. These two fixed points correspond to the self-trapping states in double-well systems [20, 21]. When the barrier is low enough, $\delta > (\Lambda - \beta)$, the two fixed points will merge into the first excited states, i.e., $z = 1$. In bottom row of Fig. 3, we show the phase spaces of the classical Hamiltonian (5) for three typical values of barrier heights.

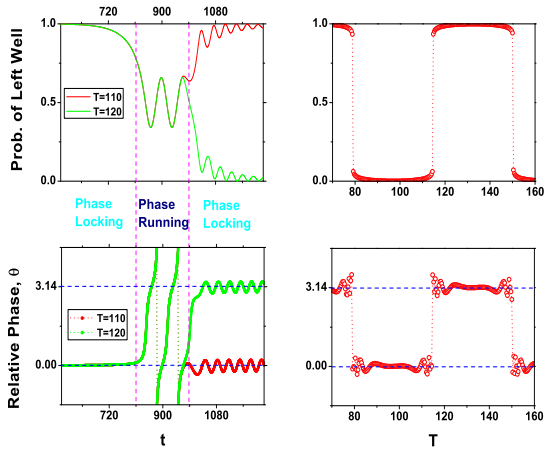


FIG. 4: Numerical results for the transfer probability and relative phase obtained by the classical system (5). The left column shows the time evolutions for $T = 110, 120$. The right column exhibits the dependence of final occupations and phases on T .

In the above process, the system is initially fully localized in the right well, i.e., initially at the fixed point $(z^*, \theta^*) = (0, 0)$. As the barrier height decreases slowly, the system evolves along the fixed point $(z^*, \theta^*) = (\frac{\delta}{\Lambda - \beta}, 0)$ until the barrier height is so low that $\delta > \Lambda - \beta$. At this time, we hold the barrier unchanged for time T . During the holding stage, the state evolves close to the first excited state with relative phase θ running. The Josephson oscillation appears during this stage. As the barrier height is raised again, the running phase orbit will drop into phase locking orbit around one of the two fixed points. Since the initial state is a fixed point, the adiabatic evolution guarantees the final state should be close to one of the two fixed points $(0, 0)$ or $(0, \pi)$ [22]. Because of the symmetry, the probability for dropping into these two phase locking regions are the same, and the period of the rectangular function is determined by the period of

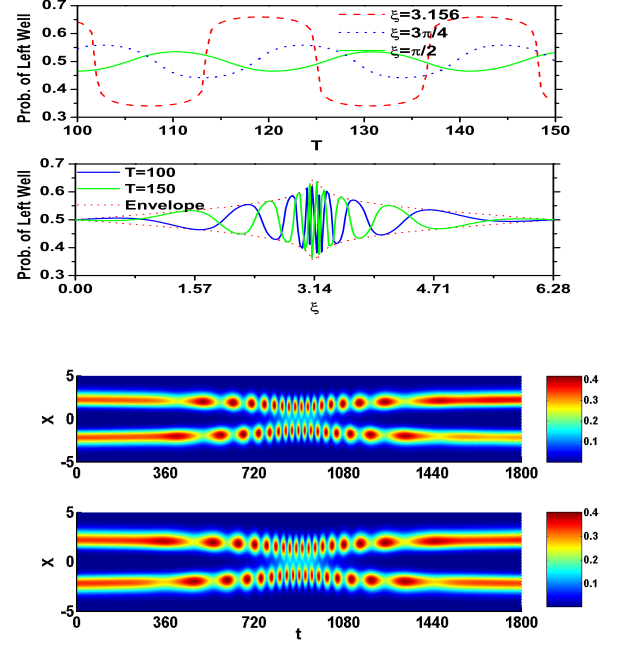


FIG. 5: The upper panel plots the final transfer probabilities of different initial relative phase ξ vs the holding time T calculated with the classical model. The second row plots the the final transfer probabilities vs different initial relative phase ξ for $T = 100$ and 150 . The dotted line (red) is the envelope for the extreme values of every ξ when changing the holding time T . The next two rows are two examples obtained by directly solving the GP equation for $\xi = 1.01\pi$ and 1.02π , respectively.

the running phase orbit when the barrier is held. From the classical model, the period can be calculated theoretically, which is $\frac{2\pi}{\sqrt{(\delta - \beta)^2 - \Lambda^2}}$ (in which the parameters are chosen as the values for the holding stage), and for the above case it is about 35, which is consistent with the above calculation. The period can be controlled by the inter-atom interaction.

In Fig. 4, using the classical Hamiltonian system (5) with the barrier U varying as showing in Fig. 1, and parameters depending on U from Fig. 3, we plot two typical probability evolutions in the left column, and the final probabilities and relative phase θ versus T in the right column. These figures show that the phenomena predicted by the GP equation can be well reproduced and understood by the classical Hamiltonian system (5).

Another interesting case is for the atoms populating evenly in two wells (e.g., ground state). For such a case, the population imbalance after the Rosen-Zener process is determined by both the holding time T and the initial phase difference ξ , the latter can be controlled with the 'phase-imprinting' technique of shining two laser beams with different intensity [23]. Fig. 5, calculated by the

Hamiltonian system (5), exhibits the final populations in the left well versus the holding time T for different ξ (the top row), and the populations versus ξ for $T = 100$ and 150 (the second row). These figures show that, after carrying out the Rosen-Zener scheme, the final population occupations of the two wells depend on the relative phase ξ as well as the holding time T . In particular, for a fixed T , the final occupations vary with the holding time ξ showing a nice interference pattern in the time domain. The interference pattern depends on the nonlinear interaction and reduces to a sinusoidal function in the absence of inter-atom interaction.

The numerical results also show that the final occupations are sensitive to the phase ξ , especially around $\xi = \pi$. Thus around the first excited state, the evolution is very sensitive to the initial condition. These results are supported by directly solving the GP equation. The bottom two rows of Fig. 5 are the density evolutions for $\xi = 1.01\pi$ and 1.03π , respectively, obtained by the GP equation, from which this sensitivity can be found.

From the above simulations, we find that the final occupations on the two wells sensitively depend on the initial conditions. Hence, we can extract the initial information from the final occupations. For example, usually, the phase difference between the condensates in two wells is measured by the spatial interference after withdrawing the barrier. Here we show that the phase difference can also be measured alternatively by interference in the time or phase domain. On the other hand, with designing a Rosen-Zener scheme, one could realize the double-well BECs with definite population imbalance and relative phase serving as coherent matter wave source used for other practical purpose.

Here, we only consider the case when the double well is symmetric. If the double well is asymmetric, the interference fringe would be sensitive to the energy bias, which affects the relative phase. The gravitational field and any acceleration can create asymmetry in the double well [24], hence, the interference fringe obtained by adiabatic Rosen-Zener interferometry can be used to measure the gravitational field or any other acceleration.

In summary, a scheme for an interferometer with the matter wave in a double-well potential serving as coherent sources is proposed. This scheme is robust and realizable with present experimental techniques. With it, the population imbalance of the atoms in two wells shows interesting interference patterns in the time domain. The fringe pattern is sensitive to the initial state, the inter-atomic interaction, and the external forces such as gravity which can change the shape of the double well. In this sense, this interferometric scheme has the potentials for precision measurements with ultracold atoms.

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